Formulation of an Imperfect Quadrilateral Doubly Curved Shell Element for Postbuckling Analysis

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Abstract

A DOUBLY curved quadrilateral shell element with 48 degrees of freedom is formulated to perform the postbuckling analysis of geometrically imperfect shells. The element surface and the geometric imperfections are specified using variable-order polynomials to represent a wide range of shell geometries and geometric imperfections. To accurately account for rigid-body modes, the Cartesian displacement components are used to express the shell displacements. A series of examples is analyzed to evaluate the performance of the element. The element can be used to analyze arbitrary shells having arbitrary imperfections.

Contents

During fabrication of shell structures, substantial deviations between the actual (fabricated) and intended shapes may occur. These deviations are generally quite localized and may significantly alter the structural behavior of these shells. The influence of geometric imperfections on the buckling behavior of shells has been the subject of much research. Koiter[1] studied the influence of initial geometric imperfections on the buckling load of axially compressed cylindrical shells. Hutchinson et al.[2] presented a combined theoretical and experimental investigation on the effects of local axisymmetric imperfections on the buckling load of axially loaded cylinders.

In previous analytic studies, the shapes of the shell structures were generally cylindrical or spherical and the imperfections were assumed to be either axisymmetric or a sinusoidal function along the entire circumference. For more complex shell structures that have an arbitrary form of imperfections, boundary, and loading conditions, numerical methods such as finite element method have their advantages. Here doubly curved quadrilateral shell element with 48 degrees of freedom (dof) is formulated for studying the geometrically nonlinear behavior of imperfect shells. The curved surface of the element and the geometric imperfections are specified using variable-order polynomials. To accurately account for rigid-body modes, Cartesian displacement components are used to express the shell displacements.

Figure 1 shows the undeformed middle surface of the shell element embedded in a Cartesian coordinate system $x^i\,(i=1,2,3)$. Cartesian coordinates of a given point are described by $x^i=f^i(\theta^1,\ \theta^2)$ in which the parameter θ^α, ($\alpha=1,2$) serve as coordinates on the surface. The two-unit base vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ and the unit normal vector $\boldsymbol{a}_3$ $(=n^i e_i)$ are defined in Fig. 1. The first and second fundamental tensors $a_{\alpha\beta}$ and $b_{\alpha\beta}$, respectively, are given as $a_{\alpha\beta}=\boldsymbol{a}_\alpha\cdot\boldsymbol{a}_\beta$ and $b_{\alpha\beta}=n^i\partial^2 f^i/\partial\theta^\alpha\partial\theta^\beta$. A point on the middle surface with the coordinates θ^α has the Cartesian coordinates $f^i(\theta^1,\ \theta^2)$ in the initial state. After deformation, the same point has the Cartesian coordinates $\bar{f}^i=f^i+u^i$, where the u^i are the Cartesian components of the displacement. In the deformed state, the first fundamental tensor becomes $\bar{a}_{\alpha\beta}=\bar{f}^i_{,\alpha}\bar{f}^i_{,\beta}$ and the curvature tensor becomes $\bar{b}_{\alpha\beta}=\bar{n}^i\bar{f}^i_{,\alpha\beta}$. The tangential strain measure $\epsilon_{\alpha\beta}$ and the curvature $\kappa_{\alpha\beta}$ are given as

$$\epsilon_{\alpha\beta}=\tfrac{1}{2}(\bar{a}_{\alpha\beta}-a_{\alpha\beta}) \tag{1}$$

$$\kappa_{\alpha\beta}=-(\bar{b}_{\alpha\beta}-b_{\alpha\beta})+\tfrac{1}{2}(b^\delta_\alpha\epsilon_{\beta\delta}+b^\delta_\beta\epsilon_{\alpha\delta}) \tag{2}$$

For the case of imperfect shell, let v^i be the Cartesian components of the imperfection at a given point on the shell surface. The tangential strain measure then becomes

$$\epsilon_{\alpha\beta}=\tfrac{1}{2}(f^i_{,\alpha}u^i_{,\beta}+f^i_{,\beta}u^i_{,\alpha}+u^i_{,\alpha}u^i_{,\beta}+v^i_{,\alpha}u^i_{,\beta}+v^i_{,\beta}u^i_{,\alpha}) \tag{3}$$

The effect of imperfections on the curvature-displacement relations are ignored. The expression for strain and curvature tensors in terms of Cartesian displacement components are given in the full paper.

The thin-shell finite element is quadrilateral in shape and has four corner nodal points as shown in Fig. 1. The Cartesian coordinates x^1, x^2, and x^3 and the imperfections v^1, v^2, and v^3 of the middle surface of the shell finite element can be described by polynomial functions of the curvilinear coordinates ξ and η with a total of N terms,

$$x^i(\xi,\eta)=\sum_{j=1}^{N}C^i_j\xi^{m_j}\eta^{n_j}\qquad i=1,2,3 \tag{4}$$

and similarly for v^i. In Eq. (4), the constants m_j and n_j define the powers of ξ and η, respectively, for the jth term. The constants C^i_j are solved based on the coordinates x^i and imperfections v^i at N selected points on the middle surface of the shell as shown in Fig. 1. The element has 12 dof at each node: u^i; $u^i_{,1}$; $u^i_{,2}$; and $u^i_{,12}$ $(i=1,2,3)$. The interpolation functions are the same as those used by Yang et al.[3]

The nonlinear effects due to large displacements and imperfections were included using an incremental formulation based on Lagrangian mode of description. If $\{q_e\}$ is the vector of the nodal displacements at a given load configuration, then the incremental strain $\{\bar{\epsilon}\}$ due to an incremental nodal displacement vector $\{\bar{q}_e\}$ can be written as

$$\{\bar{\epsilon}\}=[B_0+B_i+B_L(q_e)]\{\bar{q}_e\} \tag{5}$$

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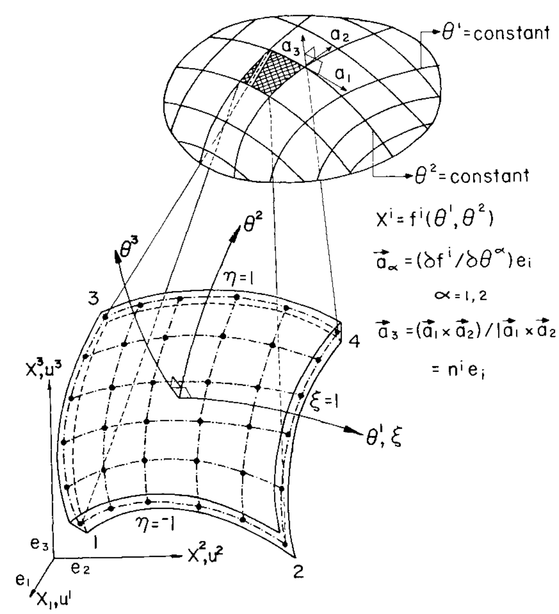

Fig. 1 Middle surface of the shell in undeformed configuration and a 48 dof doubly curved quadrilateral shell element.

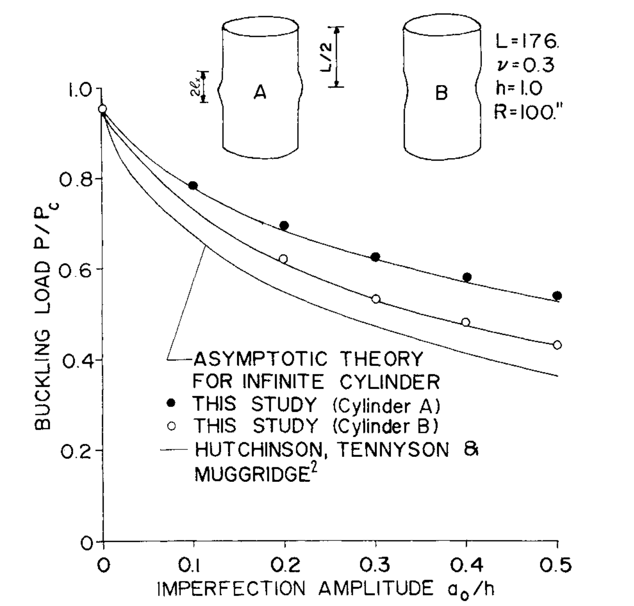

Fig. 2 Imperfection sensitivity of an axially loaded cylindrical shell with axisymmetric local imperfections.

where the strain-displacement matrix $[B_0]$ is the same as that used in linear analysis, $[B_i]$ is the matrix due to geometric imperfections, and $[B_L(q_e)]$ the matrix due to nonlinear terms. The governing equations derived using the principle of stationary potential energy are solved using Riks-Wempner constant arc method.

The imperfection sensitivity of a clamped cylindrical shell with axisymmetric local imperfections was studied. The geometry of the shell and imperfections are shown in Fig. 2. Two types of local imperfections were considered: inward and outward dimples. The stability limits were calculated using a 6 × 8 mesh to model an octant of the shell. The ratios of the buckling load to its classical Euler value were plotted for different values of imperfection amplitude in Fig. 2. For comparison purposes, analytical results obtained by Hutchinson et al.[2] are also shown in Fig. 2.

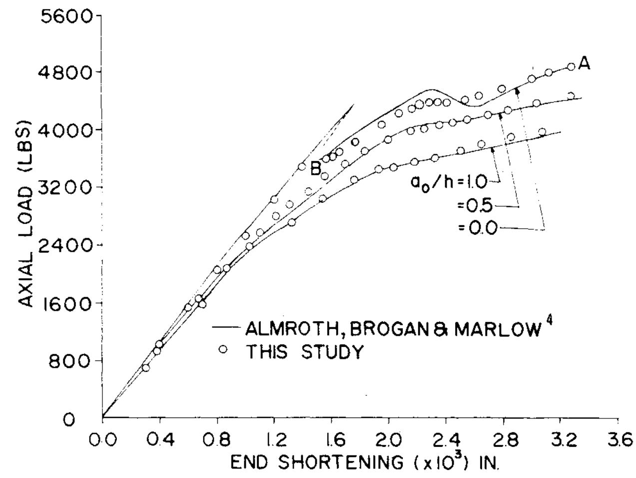

Fig. 3 Equilibrium path for elliptical cylinder.

Geometrically nonlinear analysis of a cylindrical shell with an elliptical cross section was studied under uniform end shortening. This problem was previously studied by Almroth et al.[4] using an energy-based finite difference method. In this study, an octant of the shell was modeled using a mesh of 2 × 6 elements. The load vs end shortening curves were obtained for three different values of a_0/h ratio. The present results obtained using a displacement increment algorithm are shown in Fig. 3 along with the results obtained in Ref. 4. A good agreement is seen. Using this algorithm, it was not possible to compute the equilibrium configuration in the postbuckling range for the case of perfect shell. An indirect method similar to the one used in Ref. 4 was used.

The 48 dof doubly curved quadrilateral shell element presented here is capable of modeling shells of arbitrary geometry and having arbitrary imperfections. Due to the use of Cartesian displacement components to express the shell displacement, the element can accurately account for the rigid-body modes. The presented results clearly demonstrated the validity and the accuracy of the present *imperfect* finite element. The element can be applied to shells for which a complete description of the imperfections is known from actual measurements. Recently, the authors[5] employed this element to study imperfect laminated cylindrical panels with measured imperfections and the results were compared with existing results.

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